MATHEMATICS
ESSAY PROBLEMS

Name : _____ **Participant ID :** _____

Country : _____ **Seat Number :** _____

22nd International Mathematics and Science Olympiad
Alor Setar, Kedah, Malaysia
06 October 2025

Instructions:

1. Write your name, country and Participant ID on every page of this booklet.
2. Write your detailed solution in Arabic Numerical or English in the space provided for each question in this booklet. If you need more space for your working solutions, you may use the reverse side of each page, but please write "next side continued" in the last line.
3. There are **13** questions in this paper.
4. Each question is worth 3 marks and partial credits may be awarded.
5. All diagrams are **NOT** drawn to scale. They are intended only as aids.
6. You have **90** minutes to complete this paper.
7. Use black pen or blue pen or pencil to write your answer.

Do not turn over this page until you are told to do so.

ESSAY PROBLEMS

Name: _____ Country: _____ Participant ID: _____

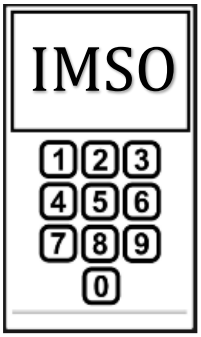
1. A four-digit number has the following properties:
   (a) It is a perfect square.
   (b) The first two digits are the same.
   (c) The last two digits are the same.
   List down all possible four-digit number(s) that satisfy the conditions.

Answer: _____

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2. Ahmad wishes to open a door secured by a four-digit code corresponding to the word **IMSO**.



Each letter is assigned a digit subject to the following conditions:

(1) The letters I and O must be odd.
(2) The letters M and S must be even.
(3) All four digits must be distinct.

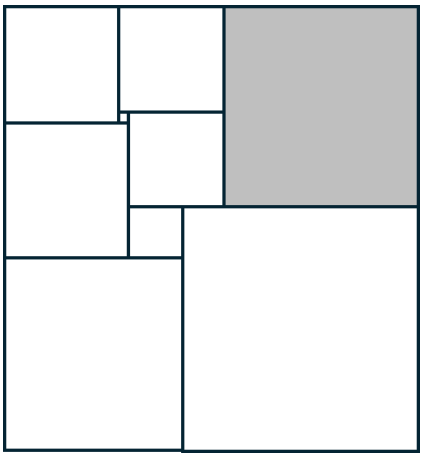
In how many ways can Ahmad enter the code?

Answer: _____ ways

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3. In the diagram below, a rectangle is composed of nine squares. Given that the area of the shaded square is 100 cm^2. What is the area, in cm^2, of the rectangle?



Answer: _____ cm^2

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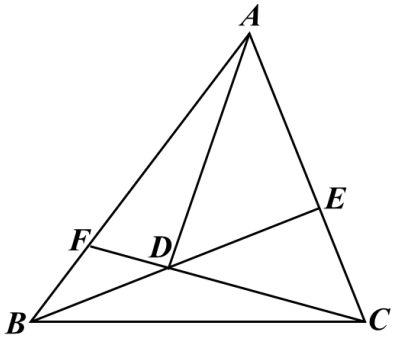
4. Consider all eight-digit integers of the form $\overline{aaaabbbb}$ which are multiples of 21. List down all possible ordered pairs (a, b).

Answer: _____

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5. In the diagram below, ABC is a triangle, where E is a point on AC and F is a point on AB. The lines BE and CF intersect at D. If the ratio of the areas of the triangles BDF and CDE is 5:16, the ratio of the areas of the triangles AFD and AED is 5:8 and the area of the triangle BCD is 18 cm^2, what is the area, in cm^2, of the triangle ABC?

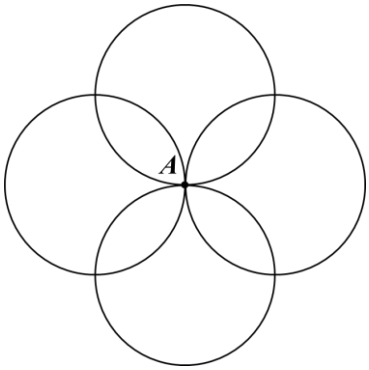


Answer: _____ cm^2

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6. Four cyclists start riding at exactly 12:00 noon. Each cyclist rides on a different circular track, where each track is exactly 1 mile long. The four cyclists ride at four different constant speeds, which are 6, 9, 12 and 15 miles per hour, respectively. All four tracks meet at the center (Point *A*). They all start together at the center and keep riding around their own track. They agree to stop when they have come back to the center together for the sixth time (counting the start at 12:00 noon as the first time). At what time do they stop?



Answer: _____ : _____ : _____ _____
Hour : *Minute* : *Second* AM/PM

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7. Let *I, M, S* and *O* be digits that are not necessarily different that satisfies

$$\overline{IMSO} = \frac{7}{4} \times \overline{OSMI}$$

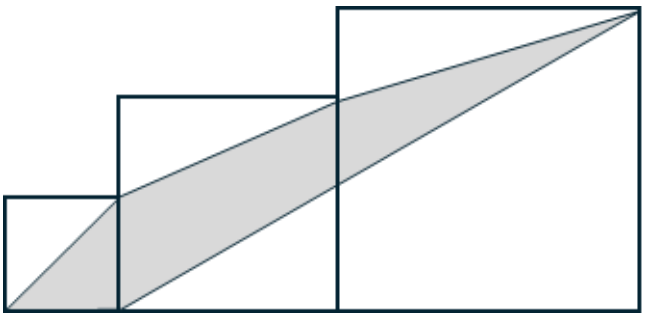
How many possible four-digit numbers $\overline{IMSO}$ are there?

Answer: _____

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8. In the diagram below, three squares with side lengths in the ratio of 2 : 3 : 5 are placed next to each other. If the area of the shaded region is 152 cm^2, then what is the perimeter, in cm, of the diagram?



Answer: _____ cm

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9. In a video game, you start the game with 10 gold coins, 10 wooden coins, 10 stone coins, 0 silver coins and 0 diamond coins.



10 Gold coins



0 Silver coins



10 Wooden coins



10 Stone coins



0 Diamond coins

There are four trading options that are available:

a.) Trading gold coin(s) to a silver coin:

The cost increases by 1 gold coin each time you do a trade.

- First trade: exchange 1 gold coin → receive 1 silver coin.
- Second trade: exchange 2 gold coins → receive 1 silver coin.
- Third trade: exchange 3 gold coins → receive 1 silver coin, and so on.

b.) Trading wooden coin(s) to silver coins:

The cost increases by 1 wooden coin each time you do a trade.

- First trade: exchange 3 wooden coins → receive 2 silver coins.
- Second trade: exchange 4 wooden coins → receive 2 silver coins.
- Third trade: exchange 5 wooden coins → receive 2 silver coins, and so on.

c.) Trading stone coins(s) to a diamond coin:

The cost increases by 1 stone each time you do a trade.

- First trade: exchange 2 stone coins → receive 1 diamond coin.
- Second trade: exchange 3 stone coins → receive 1 diamond coin.
- Third trade: exchange 4 stone coins → receive 1 diamond coin, and so on.

d.) Trading a silver coin to a diamond coin:

Fixed rate: exchange 1 silver coin → receive 1 diamond coin.

Some additional trading conditions:

- You may perform trades in any order and any number of times, provided you have enough coins at the moment of the trade.
- If you do not have enough coins to perform the required condition for a trade, then you cannot perform that trade. (You cannot partially perform a trade).
- The counters for each of the different trade types are independent with each other.

What is the maximum number of diamond coins you can obtain? Give a full and detailed explanation and describe the general strategy that achieves the maximum.

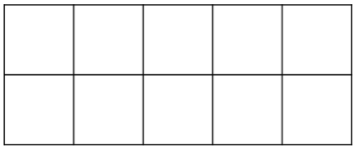
Please write down your full solution at the reverse side of this sheet.

Answer: _____ diamonds

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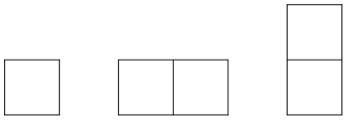
10. The diagram below shows a grid with 2 rows and 5 columns.



2 × 5 grid

The grid must be completely covered using the following types of tiles namely:

- a 1-unit tile, which occupies one space (1 × 1) or
- a 2-unit tile, which occupies two spaces and may be placed either horizontally (1 × 2) or vertically (2 × 1).



1 × 1 tile 1 × 2 tile 2 × 1 tile

How many different ways are there to completely cover the entire 2 × 5 grid without overlapping?

Answer: _____ ways

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11. For each positive integer n, let $S(n)$ be the sum of its digits and $P(n)$ be the product of its digits.

For example, if $n = 2025$,

$S(2025) = 2 + 0 + 2 + 5 = 9$ and $P(2025) = 2 \times 0 \times 2 \times 5 = 0$.

List down all the possible three-digit numbers n that satisfy $n = S(n) \times P(n)$.

Answer: _____

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12. Twenty students participated in a chess tournament, where each student played every other student exactly twice.
A student earns 3 points for a win, 1 point for a draw and 0 points for a loss.
What is the maximum number of students that can have at least 90 points at the end of the tournament?

Answer: _____ students

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13. Let a, b, and c be distinct positive integers satisfying $a^b \times b^a = (a \times b)^c$. What is the smallest possible value of $a + b + c$ across all valid triples (a, b, c)?

Answer: _____